The Mayergoyz vector Preisach-type model in analysis of irreversibilities in financial systems

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The following two trends have emerged in the modelling of macroeconomic and financial processes.

- *Interest in equilibria and the role of shocks in their formation.* Standard economic analysis assumes that economic equilibria are *homeostatic*, in that the reversal or removal of a temporary shock will be accompanied by a return to the original equilibrium. This assumption, however, rarely holds in economic systems, and in financial systems it is just as unreliable. Keynes [1] answered the question "are economic systems self-adjusting?" in the negative. The persistent effect of temporary shocks are known as *heterostasis*, with a range of equilibria available, the selection being realised by the history of such temporary shocks. Permanent effects of a temporary stimuli is a characteristic of systems with hysteresis, see [2], [3].
- *Interest in dynamics in zones separated from equilibria.* It is increasingly accepted that economic and financial systems operate in zones separated from equilibria, and that the analysis of their behaviour in such zones is important. Some classical models, when studied in this way, yield very interesting results, for example the finding of chaos in the classical Kaldor model [4] and the Kaldor model with hysteresis [5].

The study of these two trends, which has largely been done separately, have recently been brought together. A new approach, suggested in [6], based on a special type of differential-operator equation of the form

$$\dot{y} = f(t, x), \qquad y(t) = (H\left[\eta(t_0)\right] x)\,(t).$$

where H denotes a particular hysteresis operator. In [6], [7], this equation was considered with H chosen to be a particular Preisach nonlinearity. In terms of financial systems, where a number of factors make up the "input" of the system, the one-dimensional nature of this system is drawback. We will consider a multi-dimensional (vector) analogue of this model, using the Isaak Mayergoyz vector Preisach-type nonlinearity [8].

We consider the modelling of financial markets, which exhibit three properties that suggest that investigating the presence of hysteresis would be worthwhile. Firstly, the effects of shocks in these systems are persistent. Secondly, individual market entry/exit decisions are not reversible, which suggests the use of non-ideal relays in models of the behaviour. And finally, the agents that make up the financial market are heterogeneous.

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Numerical models representing agents using one-dimensional non-ideal relays have produced behaviour similar to the stylised facts seen in financial markets, such as "fat-tails" in the distribution of price returns, [9], [10]. We use the Mayergoyz vector Preisach-type model to investigate situations in which agents respond to two or more input streams to examine if such hysteresis effects can be observed in financial market prices. An example is in currency markets (or currency futures markets) where inputs include relative interest rate differentials and trading volume.

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